

Hadron resonances with coexistence of different natures

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Abstract. We discuss coexistence/mixing of different natures of hadronic composite (molecule) and elementary (quark-intrinsic) ones in hadron resonances. The discussions here are based on our previous publications on the origin of hadron resonances [1], exotic \bar{D} meson-nucleons as hadronic composites containing one anti-heavy quark [2], and the study of a_1 as a typical example to show explicitly the mixing of the two different natures [3]. In all cases, interactions are derived from the chiral dynamics of the light flavor sector. These interactions generate in various cases hadronic composite/molecule states, serving varieties of structure beyond the conventional quark model.

1 Introduction

One of recent activities in hadron physics has been motivated by the observations of exotic hadrons. The candidate of the truly exotic five quark state Θ^+ is now under further investigations [4,5], while the existence and its nature of $X(3872)$ is being established [6]. Very recently yet strong candidates of the true tetraquark states in the bottom sector have been observed near the $B\bar{B}^*$ ($B^*\bar{B}$) and $B^*\bar{B}^*$ thresholds [7].

In general, multi-quark components are expected to exist also in the conventional hadron resonances. Because a typical excitation energy of hadron resonances is of order half GeV, additional pair of quark and anti-quark may be created, forming a multi-quark component in the resonance state. A multi-quark component may rearrange itself such that its energy takes the minimum. Among such correlations diquark, triquark and hadronic ones are widely assumed [8,9]. The former two form colored clusters existing only inside hadrons, while the latter may develop a molecular like structure; two (or more in general) hadrons are loosely bound while each hadron keeps its identity. In other words, such states have a larger spatial size as compared to their constituent hadrons.

Here we consider several hadronic molecules, or in what follows we shall call them hadronic composite, because we know their interactions better than the interaction between colored clusters. We expect that such states should be seen near the threshold region of the constituent hadrons, though their appearance is not trivial, depending on the nature of the relevant hadron interactions. If the interaction is too strong, they may form a strongly bound system which cannot keep the identity of a constituent hadron.

The hadronic composite is another candidate of exotic hadrons in wider sense, although we have not yet confirmed any example of such hadronic composites except for atomic nuclei. At present, almost all hadrons listed in the particle data are explained regarding their quantum numbers by the

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constituent quark model where mesons and baryons are described by $q\bar{q}$ and qqq , respectively. It is, however, widely expected that some of hadrons have different structure from those of the constituent quark model, such as $\Lambda(1405)$ [10,11] and scalar mesons $\sigma(600)$, $f_0(980)$, $a_0(980)$ [12,13]. They are rather generated by coupled channels of $\bar{K}N-\pi\Sigma$ [14,15] and $\pi\pi-\bar{K}K$, respectively.

Once we have such hadronic descriptions with reliable interactions, we solve the scattering problem, where resonance states appear as poles of the scattering amplitude. This is how hadronic composites are described and are dynamically generated.

In general, however, we need explicit introduction of intrinsic degrees of freedom which is not in the original theory of hadronic composites to explain experimental data. This is natural because many resonances are described by the quark model. In the examples we discuss here, the necessity is associated with the structure of the constituent hadrons having a smaller size than the hadronic composites. Therefore, we assume two hierarchies discriminated by spatial size; hadrons described by the quark model with a smaller size and hadronic composites with a larger size. Following Weinberg [16], we call the quark model states elementary, in the sense that they are not easily described as composite states of hadrons.

In this report, we discuss three topics. One is the introduction of the elementary component in a hadronic model based on the scattering of a Nambu-Goldstone boson in the chiral unitary approach [1]. The interaction for the Nambu-Goldstone bosons is given by the Weinberg-Tomozawa theorem. This picture has been extensively studied for kaon dynamics which seems to have an ideal ground to form hadronic composites. Second we discuss hadronic composites in the heavy quark sector where we predict several bound and resonant states of \bar{D} and B mesons [2,17]. The interaction is mediated by the pion, which becomes crucial when the coupled channel effect becomes important though the mixing of S and D waves due to the tensor component of the pion exchange force. The mechanism is similar to the one for the deuteron where the one pion exchange force plays a crucial role for the binding of the deuteron [18]. For the \bar{D} and B meson cases, the degeneracy between $\bar{D}-\bar{D}^*$ and $B-B^*$ mesons are important, which is indeed the case in the limit of heavy quark mass. In the third topic we discuss the coexistence of the two different structures, the hadronic composite and elementary components [3]. An analysis is made for the axial vector meson a_1 where well established chiral Lagrangians with vector mesons are available.

2 Hadronic composites in the chiral unitary model

The chiral unitary model has been extensively used for the study of hadron resonances as dynamically generated states [11,14,15,12,13]. It is based on the chiral perturbation theory and the basic amplitudes at low energies are provided by the Weinberg-Tomozawa (WT) interaction for the S-wave scattering of a Nambu-Goldstone boson from a baryon (in general, matter). This low energy amplitude is then unitarized to extend into the resonance region. Typical successful cases are for $\Lambda(1405)$ and scalar mesons as resonances as $\bar{K}N-\pi\Sigma$ [14,15] and $\pi\pi-\bar{K}K$ scattering. The WT amplitude is given by the Lagrangian at the tree level

$$L_{WT} = \frac{1}{f_\pi^2} V_c^\mu f_{abc} (\partial_\mu \phi^a) \phi^b \quad (1)$$

where f_π is the pion decay constant, f_{abc} are the structure constant of flavor SU(3). The octet NG boson field is denoted by ϕ^a , and V_c^μ are for the octet components of the vector current of the target particle.

The amplitude is then unitarized by using (1) as a potential for the Schrödinger equation. We can solve it and derive the scattering matrix T ;

$$T(E) = V_{WT} + V_{WT} G(E) T \quad (2)$$

where the Weinberg-Tomozawa interaction V_{WT} is obtained from the Lagrangian (1). By using the pointlike (separable) nature of the interaction V_{WT} as well as the on-shell factorization [11], the two-body propagator $G(E)$ is separated from the integral equation, and is given in the form of a one-loop integral, which can be performed analytically. However, precisely due to the pointlike nature, the loop

integral diverges and a suitable renormalization is needed to obtain a finite result. Two methods are often used, one is the dimensional regularization with removing the divergent terms and introducing a subtraction (renormalization) constant a . Another is to express it as an integral over three dimensional momentum and perform the integral up to a cutoff momentum Λ , which is equivalent to the sum over intermediate states in the second order perturbation theory:

$$G(E) \sim \sum_n \frac{1}{E - E_n} \quad (3)$$

Mathematically, this sum runs up to infinitely large momentum, while a finite size structure of the constituent hadrons may render a cutoff, giving a finite result. In this case due to its structure of (3), the G-function takes a negative value for energies below the threshold ($E < E_n$ for all n).

In Ref. [1], this property was used to introduce the natural condition for the G-function calculated in the dimensional regularization, which determines the natural subtraction constant $a = a_{\text{natural}}$. If resonance properties are reproduced by using this natural value, we can say that the resonance is dynamically generated and is described as a hadronic composite. It was shown in Ref. [1] that $\Lambda(1405)$ is a typical resonance of such. The natural value a_{natural} for this case corresponds to the cutoff $\Lambda \sim 600$ MeV, consistent with $\bar{K}N$ (loosely) bound state with intrinsic size of constituent hadrons of order of 0.5 fm.

In many cases, however, resonance properties are not reproduced by using the natural subtraction constant, where we need to fix it at $a \neq a_{\text{natural}}$. It was shown that this was indeed the case for $N(1535)$ [1]. What is interesting, however, that when $a \neq a_{\text{natural}}$, the difference $\Delta a = a - a_{\text{natural}}$ introduces a pole-like interaction additionally:

$$T(E) = \frac{1}{V_{WT}^{-1} - G(E; a)} = \frac{1}{V_{\text{eff}}^{-1} - G(E; a_{\text{natural}})} \quad (4)$$

where

$$V_{\text{eff}} = V_{WT} + \frac{C}{2f_\pi^2} \frac{(E - M)^2}{E - M_{\text{eff}}^2}, \quad M_{\text{eff}} = M - \frac{16\pi^2 f_\pi^2}{CM\Delta a} \quad (5)$$

In these equations, M denotes the mass of the target particle, and C is a constant for the Weinberg-Tomozawa interaction. The additional piece in the interaction may be then interpreted as the propagation of an elementary particle which is not in the original model space of the two-body scattering, just like the CDD pole [19]. In this way, we can see that resonances, in general, are mixture of the hadronic composite and elementary components. More detailed analysis for the compositeness or elementariness of a particle is performed in Ref. [20].

3 \bar{D} molecule as a pentaquark baryon

As a good example of hadronic composite, we discuss in this section heavy quark systems of $\bar{D}N$ and BN . The minimal quark content of these states is $\bar{Q}qqqq$ ($Q = b, c; q = u, d$), and therefore, they are analogues of the pentaquark $\Theta^+ \sim \bar{s}uudd$ with one heavy anti-quark. An interesting feature in the heavy quark sector is the heavy quark symmetry which leads to the degeneracy of spin 0 and 1 particle; \bar{D} and \bar{D}^* , and B and B^* .

From now on let us introduce the notation P for the pseudoscalar meson, and P^* for the vector meson. Because of the degeneracy, the channel coupling of P^*N and PN becomes important. This contrasts to the light flavor system, where the mass difference of K^* and K , and that of ρ and π are much larger than that for the charm and bottom sectors, as summarized in Table 1. In this case the pion coupling at the vertex $P^*P\pi$ and $NN\pi$ allows one pion exchange potential as shown in Fig. 1. A characteristic feature of the one-pion exchange is then the tensor force as in the nuclear system; for instance, it is well known that the tensor force causes the channel coupling of S and D waves leading to a strong attraction for the deuteron. The same mechanism applies to the heavy meson system through

the transition $P^*N \rightarrow PN$. We note that the $P^*P\pi$ coupling has the structure of $\mathbf{s} \cdot \mathbf{q}$ where \mathbf{s} is a (half of) spin operator and \mathbf{q} the momentum carried by the pion. For the case of $NN\pi$, \mathbf{s} is the Pauli matrix, while for $P^*P\pi$ it is the spin transition operator between spin 1 and 0 states.

Table 1. Mass differences of pseudoscalar and vector mesons.

| $m_\rho - m_\pi$ | $m_{K^*} - m_K$ | $m_{\bar{D}^*} - m_{\bar{D}}$ | $m_{B^*} - m_B$ |
|------------------|-----------------|-------------------------------|-----------------|
| 630 MeV | 400 MeV | 140 MeV | 45 MeV |

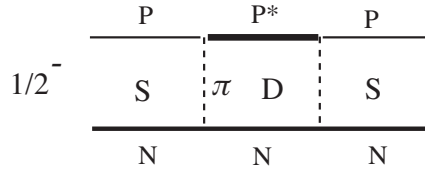


Fig. 1. Pion exchange between the P (P^*) meson and the nucleon (N).

The above idea has been proposed and tested in [17], and further elaborated in [2]. Then it turns out that precisely due to the mixing mechanism, the system develops a low lying bound state, in this case in $J^P = 1/2^-$. Furthermore, a resonance state was found for $J^P = 3/2^-$. The necessary coupled channels are shown in Table 2. In Ref. [2], a short range interaction mediated by ρ and ω exchanges was also included, where it was shown that these meson exchanges play only minor role as discussed below (Table 3).

Table 2. Various coupled channels for a given quantum number J^P for negative parity $P = -1$.

| J^P | channels | | | |
|---------|-----------------|-------------------|-------------------|-------------------|
| $1/2^-$ | $PN(^2S_{1/2})$ | $P^*N(^2S_{1/2})$ | $P^*N(^4D_{1/2})$ | |
| $3/2^-$ | $PN(^2D_{3/2})$ | $P^*N(^2D_{3/2})$ | $P^*N(^4D_{3/2})$ | $P^*N(^4S_{3/2})$ |

The bound state properties are summarized in Table 3 for the two cases when the one pion (π) and π, ρ, ω exchange potentials are employed. For the $\bar{D}N$ system, the binding energy and spatial size are similar to those of the deuteron. The system is then indeed loosely bound and consistent with a picture of hadronic composite. For the heavier system of the bottom quark, the binding energy becomes larger due to stronger SD coupled channel effect and to less kinetic energy of heavier mass particle. Yet the size is larger than 1 fm, and we may regard them as hadronic composites.

Table 3. The binding energies and the root mean square radii of the $J^P = 1/2^-$ bound states.

| | $\bar{D}N(\pi)$ | $\bar{D}N(\pi\rho\omega)$ | $BN(\pi)$ | $BN(\pi\rho\omega)$ |
|------------------------------|-----------------|---------------------------|-----------|---------------------|
| Binding energy [MeV] | 1.60 | 2.14 | 19.50 | 23.04 |
| Root mean square radius [fm] | 3.5 | 3.2 | 1.3 | 1.2 |

In Ref. [2], resonance states are also found for $J^P = 3/2^-$ for both charm and bottom sectors. This is the so called Feshbach resonance, where a would-be bound state becomes a resonance when the

coupling to the open PN channel is turned on. In Fig. 2, phase shifts of PN scatterings are shown as functions of the scattering energy from the threshold. Phase shifts indeed cross at $\delta = \pi/2$ sharply indicating the presence of a narrow resonance. We have extracted resonance energies and the corresponding width as summarized in Table 4. The narrow width can be understood by the D-wave nature of the decay channel of PN and small phase space volume.

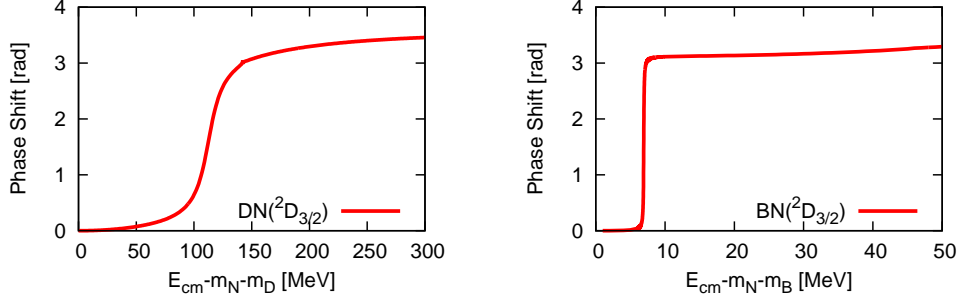


Fig. 2. Phase shifts of the $\bar{D}N$ and BN scattering for $J^P = 3/2^-$ with $I = 0$.

Table 4. The resonance energy and decay width in the $J^P = 3/2^-$ channel with π, ρ, ω exchange potential.

| | $E_{re} - m_N - m_P$ [MeV] | E_{re} [MeV] | Decay width [MeV] |
|------------|----------------------------|----------------|-----------------------|
| $\bar{D}N$ | 113.19 | 2919.09 | 17.72 |
| BN | 6.93 | 6224.83 | 9.46×10^{-2} |

4 Coexistence of different natures in axial vector meson a_1

In this section, we now discuss mixing of wave functions of different natures. The motivation was already discussed in the previous section 2, where the subtraction constant introduced an elementary component in the hadronic description of resonances. Here, we explicitly consider such a problem by using a suitable model for the hadronic description as well as for the elementary one.

As an example, we study the axial vector meson $a_1(1260)$ [3]. Conventionally a_1 has been considered as a chiral partner of the vector ρ meson, which can be explicitly realized by a $\bar{q}q$ meson [21,22]. Recently, it has been also dynamically generated as a resonance of π - ρ scattering and various properties have been investigated [23,24]. In fact, the chiral Lagrangian of ρ and a_1 contains the both aspects of a_1 . For instance, one can employ the hidden local symmetry approach for the vector mesons [25],

$$\begin{aligned}
 L_{\pi\rho a_1}^{HLS} = & +2f_\pi^2 \text{tr} A_\mu^2 - \frac{1}{2} \text{tr} (V_{\mu\nu}^2 + A_{\mu\nu}^2) \\
 & + 2f_\pi^2 g^2 \text{tr} \left(V_\mu + \frac{1}{g} v_\mu \right)^2 + 2f_\pi^2 g^2 \text{tr} \left(A_\mu + \frac{1}{g} a_\mu \right)^2
 \end{aligned} \tag{6}$$

where the vector and axial vector currents of the pion field are defined by

$$v_\mu = \frac{i}{2} (\partial_\mu \xi^\dagger \xi + \partial_\mu \xi \xi^\dagger), \quad a_\mu = \frac{i}{2} (\partial_\mu \xi^\dagger \xi - \partial_\mu \xi \xi^\dagger) \tag{7}$$

$$\xi = \exp(i\tau \cdot \pi / (2f_\pi)) \equiv \exp(i\pi / (2f_\pi)) \tag{8}$$

In Eq. (6) symbols V and A are for the ρ and a_1 mesons, respectively. It is shown that the Lagrangian (6) can be derived from the extended NJL model, where the mesons are constructed by $\bar{q}q$, and are considered as elementary particles [21,22]. Recently, a similar Lagrangian was derived from a holographic model of the string theory for QCD [26].

The Lagrangian contains sufficient ingredients for the present discussion of a_1 ; (1) elementary a_1 manifestly, (2) the coupling of the elementary a_1 to π and ρ , and (3) the $\pi\rho$ - $\pi\rho$ interaction for the dynamical generation of a_1 as a composite state. Hence, now the problem is to solve a coupled channel system for the elementary a_1 and the $\pi\rho$ channels. In the literature, full flavor SU(3) coupled channels are included [23,24]. However, essential properties of a_1 are reproduced by the single channel of $\pi\rho$, and so we consider here only this channel.

To proceed, it is convenient to introduce the Hamiltonian in the coupled channel form;

$$H = \begin{pmatrix} H_{\pi\rho} + V_{WT} & g \\ g & H_e \end{pmatrix} \quad (9)$$

where $H_{\pi\rho}$ is the free $\pi\rho$ Hamiltonian, H_e for the elementary a_1 which is simply its mass in the center of mass frame, and g is the coupling of the elementary a_1 to $\pi\rho$. By introducing a two-component wave function $\psi = (\psi_{\pi\rho}, \psi_e)$, we have coupled channel equations

$$\begin{aligned} (H_{\pi\rho} + V_{WT})\psi_{\pi\rho} + g\psi_e &= E\psi_{\pi\rho} \\ g\psi_{\pi\rho} + m_e\psi_e &= E\psi_e \end{aligned} \quad (10)$$

These equations are solved to obtain the $\pi\rho \rightarrow \pi\rho$ scattering T matrix in the form

$$T_{\pi\rho \rightarrow \pi\rho} = (g_R, g) \frac{1}{\begin{pmatrix} g_R(V_{WT}^{-1} - G_{\pi\rho})g_R & g_R G_{\pi\rho} g \\ g G_{\pi\rho} g_R & G_e^{-1} - g G_{\pi\rho} g \end{pmatrix}} \begin{pmatrix} g_R \\ g \end{pmatrix} \quad (11)$$

Here the Green's functions are defined by $G_{\pi\rho} = 1/(E - H_{\pi\rho})$ and $G_e = 1/(E - H_e)$, and g_R the $\pi\rho$ coupling to the composite a_1 which is defined by the pole generated by the Weinberg-Tomozawa interaction V_{WT} as

$$T_{WT} = \frac{1}{V_{WT}^{-1} - G_{\pi\rho}} \equiv g_R \frac{1}{E - M_{WT}} g_R \quad (12)$$

We note that the basis states representing the Hamiltonian (9) are the composite a_1 and the elementary one with renormalization by the self-energy $gG_{\pi\rho}g$. There are some subtleties in, for instance, energy dependence in various coupling constants, however. Detailed discussions are given in Ref. [3].

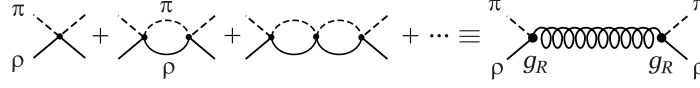
It is helpful to see the diagrammatic interpretation of this scattering matrix (11) as shown in Fig. 3. An important point here is that by expressing the T matrix as in (11), the physics behind it has become clear; the scattering occurs going through resonance states as intermediate states as described by the composite and elementary a_1 's. By diagonalizing the 2×2 matrix in the denominator (11), we obtain the physical poles.

Various properties of the physical a_1 's are discussed in Ref. [3]. Here we would like to summarize some of them.

- When $g \rightarrow 0$, the Hamiltonian is just the sum of the decoupled components of the $\pi\rho$ system for the composite a_1 and the elementary a_1 . The locations of these poles are $M_1 \sim 1112 - 221i$ and $M_2 \sim 1189 - 0i$, and determines the model basis for the discussions below. The latter is the mass of the a_1 given by the original Lagrangian (6).
- By varying the coupling strength from zero to the physical value of g , the two poles deviate from the original locations, and at the physical g they reach $M_1 \sim 1033 - 107i$ and $M_2 \sim 1728 - 313i$. Hence the original composite pole approaches the real axis, while the elementary one moves deeply into the imaginary region on the complex energy plane with much larger real value than the observed physical mass of $a_1 \sim 1260$ MeV. Therefore, the observed spectrum of a_1 is dominantly described by the lower pole which is originally of composite nature, though the agreement is not perfect.

- At the physical point, the mixing rates of the composite and elementary components are almost comparative for the pole which is originally composite, while the pole which is originally elementary is also dominated by the composite component (though these estimations are only qualitative, because these states appear as resonances and the meaning of the mixing ratio is not well defined as in the case of bound states). In this way, the physical poles are indeed mixture of the two components, and the mixing rates depend on details of the dynamics, in particular on hadron dynamics.

11 component
composite a_1



12, 21 components
composite - elementary
transition



22 component
elementary a_1

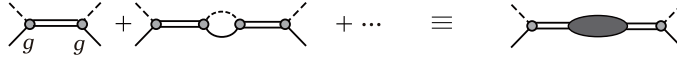


Fig. 3. Diagrammatic interpretation of various components of the effective Hamiltonian, the denominator of (11).

We have also studied the resonance properties by changing the color number N_c [27]. We find that at $N_c = 3$, the coupling of the elementary a_1 to $\pi\rho$ (g) and the composite a_1 generated by V_{WT} play an essential role to derive the properties of physical a_1 , while these terms are of higher order in $1/N_c$ and irrelevant in the limit $N_c \rightarrow \infty$, where only $\bar{q}q$ mesons survive. We have discussed geometrical aspects of the mixing nature in particular as function of the color number N_c in Ref. [27].

5 Summary

In this report, we have discussed hadron resonances starting from a picture of quark rearrangement for multi-quark systems. In general, they develop different configurations depending on the properties of the inter-quark forces, a situation much like atomic nuclei with, for instance, alpha cluster formation. As one of most plausible scenarios, we focus on configurations made by two hadronic clusters, a hadronic composite as strong candidate of non-standard hadrons which are explained by the conventional quark model.

In addition to well known examples of $\Lambda(1405)$ and sigma mesons, we discussed a heavy quark system of exotic quantum number as a strong candidate of hadronic composite. What is interesting here is that the tensor force plays a crucial role just like the mechanism in the deuteron. Precisely because of this in the literature a mesonic correspondent, for instance $X(3872)$ was called deuson [28]. We find indeed a bound and resonant states in $J^P = 1/2^-$ and $3/2^-$, respectively.

Finally we have studied mixture of different configurations in hadron structure, where an example was shown for the axial vector meson a_1 . By solving the two body scattering equation, we have derived a quantum mechanical problem of two levels with the basis of the composite and elementary a_1 . This analysis implies that there are cases where hadronic components become important for the physical hadrons.

The hadron resonances which are dominated by hadronic composites are not yet experimentally confirmed. With its loosely bound nature and with various configurations than the quark model can provide, further theoretical and experimental studies are expected. Related to the twin states of Z_b found at Belle [7], more analogous states are theoretically expected to exist [29,30]. Perhaps interesting

features of the strong interaction should be seen near the threshold where a new flavor opens and still non-perturbative dynamics plays important roles to generate hadrons of different natures.

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References

1. T. Hyodo, D. Jido and A. Hosaka, Phys. Rev. C **78**, 025203 (2008) [arXiv:0803.2550 [nucl-th]].
2. Y. Yamaguchi, S. Ohkoda, S. Yasui and A. Hosaka, Phys. Rev. D **84**, 014032 (2011) [arXiv:1105.0734 [hep-ph]].
3. H. Nagahiro, K. Nawa, S. Ozaki, D. Jido and A. Hosaka, Phys. Rev. D **83**, 111504 (2011) [arXiv:1101.3623 [hep-ph]].
4. T. Nakano *et al.* [LEPS Collaboration], Phys. Rev. Lett. **91**, 012002 (2003) [arXiv:hep-ex/0301020].
5. T. Nakano *et al.* [LEPS Collaboration], Phys. Rev. C **79**, 025210 (2009) [arXiv:0812.1035 [nucl-ex]].
6. V. Bhardwaj *et al.* [Belle Collaboration], arXiv:1105.0177 [hep-ex].
7. I. Adachi *et al.* [Belle Collaboration], arXiv:1105.4583 [hep-ex].
8. R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. **91**, 232003 (2003) [arXiv:hep-ph/0307341].
9. M. Karliner and H. J. Lipkin, Phys. Lett. B **575**, 249 (2003) [arXiv:hep-ph/0402260].
10. N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A **594** (1995) 325.
11. E. Oset and A. Ramos, Nucl. Phys. A **635**, 99 (1998) [arXiv:nucl-th/9711022].
12. J. A. Oller and E. Oset, Nucl. Phys. A **620**, 438 (1997) [Erratum-ibid. A **652**, 407 (1999)] [arXiv:hep-ph/9702314].
13. J. R. Pelaez, Phys. Rev. Lett. **92**, 102001 (2004) [arXiv:hep-ph/0309292].
14. D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, Nucl. Phys. A **725**, 181 (2003) [arXiv:nucl-th/0303062].
15. T. Hyodo and W. Weise, Phys. Rev. C **77**, 035204 (2008) [arXiv:0712.1613 [nucl-th]].
16. S. Weinberg, Phys. Rev. **130**, 776 (1963).
17. S. Yasui and K. Sudoh, Phys. Rev. D **80**, 034008 (2009) [arXiv:0906.1452 [hep-ph]].
18. T. Ericson and W. Weise, ”Pions and Nuclei”, Chapter 3, Clarendon Press, Oxford (1988).
19. L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 453 (1956).
20. T. Hyodo, D. Jido and A. Hosaka, arXiv:1108.5524 [nucl-th].
21. M. Wakamatsu and W. Weise, Z. Phys. A **331**, 173 (1988).
22. A. Hosaka, Phys. Lett. B **244**, 363 (1990).
23. L. Roca, E. Oset and J. Singh, Phys. Rev. D **72**, 014002 (2005) [arXiv:hep-ph/0503273].
24. H. Nagahiro, L. Roca, A. Hosaka and E. Oset, Phys. Rev. D **79**, 014015 (2009) [arXiv:0809.0943 [hep-ph]].
25. M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985).
26. T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141].
27. K. Nawa, S. Ozaki, H. Nagahiro, D. Jido and A. Hosaka, arXiv:1109.0426 [hep-ph].
28. N. A. Tornqvist, Phys. Lett. B **590**, 209 (2004) [arXiv:hep-ph/0402237].
29. M. B. Voloshin, arXiv:1105.5829 [hep-ph].
30. S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh and A. Hosaka, in preparation.